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PUMPING INDUCED SETTLEMENT OF AQUIFERS

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Pumping Induced Settlement of Aquifers

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Many cities around the world have experienced settlement resulting from water withdrawal in aquifers located beneath the community. Most commonly these communities are located on unconsolidated clastic sediments laid down in alluvial, lacustrine, or shallow marine environments. If unconsolidated saturated silts and clays are present above and interbedded with a confined sand and gravel aquifer, pumping from the aquifer can lead to a pore-pressure reduction in the silts and clays and change the state of stress in the aquifer through artesian head decline. This causes settlement in the aquifer and settlement in the overlying and interbedded silts and clays, which often leads to surface subsidence.

The settlement can be modeled by relating the mass balance equations describing groundwater flow to the one dimensional consolidation equation.

The governing equation of Mass Balance for three dimensional flow in a block of aquifer media of unit size can be expressed as

$$\frac{\partial}{\partial x}\left[K_x \frac{\partial \phi}{\partial x}\right]+\frac{\partial}{\partial y}\left[K_y \frac{\partial \phi}{\partial y}\right]+\frac{\partial}{\partial z}\left[K_z \frac{\partial \phi}{\partial z}\right]+S''=\frac{\partial S_a n}{\partial t} \qquad \text{Eq. 1}$$

where the terms on the left hand side represents the flow through a block of aquifer media and the right hand side represents rate at which the volume of water changes inside the element. The two terms in the numerator of the right hand side are the degree of saturation S_a and the porosity n. Together they represent the volume of water in a unit volume of material whether it is aquifer or overburden.

It is not necessary to describe the exact nature of the flow, but the extraction of the water through pumping indicates that the flow through each unit volume is unbalanced. Therefore, there is a change in the amount of water stored inside the element, or, the product $S_a n$ changes with time. The change in storage can be related to the change in volume of the unit volume.

The rate term in Eq. 1 can be expanded by using the chain rule,

$$\frac{\partial S_a n}{\partial t}=S_a \frac{\partial n}{\partial t}+n\frac{\partial S_a}{\partial t}. \qquad \text{Eq. 2}$$

This can converted from a partial derivative to an algebraic form by using the delta approximation,

$$\frac{\partial S_a n}{\partial t} = S_a \frac{\partial n}{\partial t} + n \frac{\partial S_a}{\partial t} \cong S_a \frac{\Delta n}{\Delta t} + n \frac{\Delta S_a}{\Delta t} \qquad \text{Eq. 3}$$

In this form, the terms can be manipulated using standard algebra. For example, the porosity n is related to the void ratio e by the simple equation

$$n = \frac{e}{1+e} \qquad \text{Eq. 4}$$

and the change in porosity is equal to

$$\Delta n = \frac{\Delta e}{1+e} \qquad \text{Eq. 5}$$

Therefore, Equations 2 and 3 can be rewritten

$$\frac{\partial S_a n}{\partial t} \cong \frac{S_a}{1+e} \frac{\Delta e}{\Delta t} + \frac{e}{1+e} \frac{\Delta S_a}{\Delta t} \qquad \text{Eq. 6}$$

A unit volume of soil can be divided into two parts, the solid material and the void volume which contains the water and gases, if available. This can be shown in a phase diagram as in Fig. 1, Before Settlement. After settlement has occurred, the phase diagram on the right shows that the volume of solid remains constant while the volume of void changes. Thus, the change in total volume is the same as the change in volume of void.

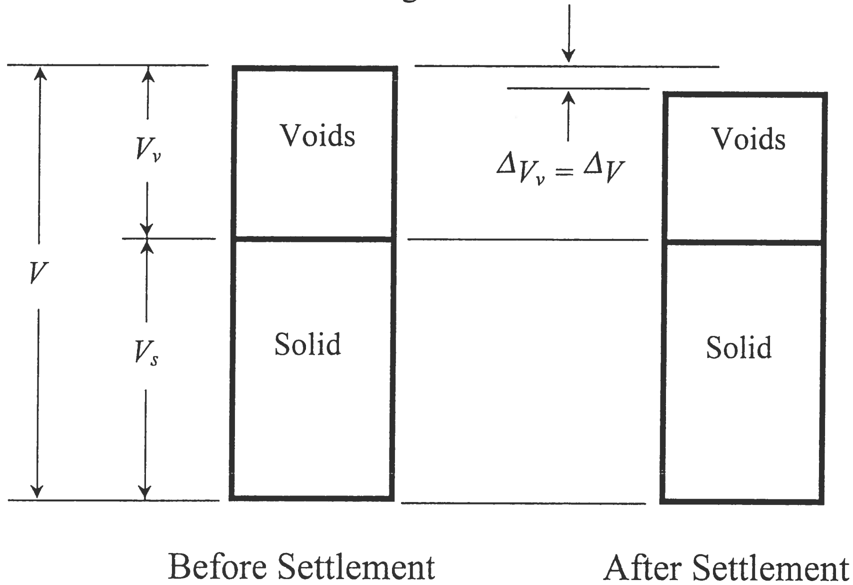

Figure 1 - Phase Diagrams Before and After Settlement

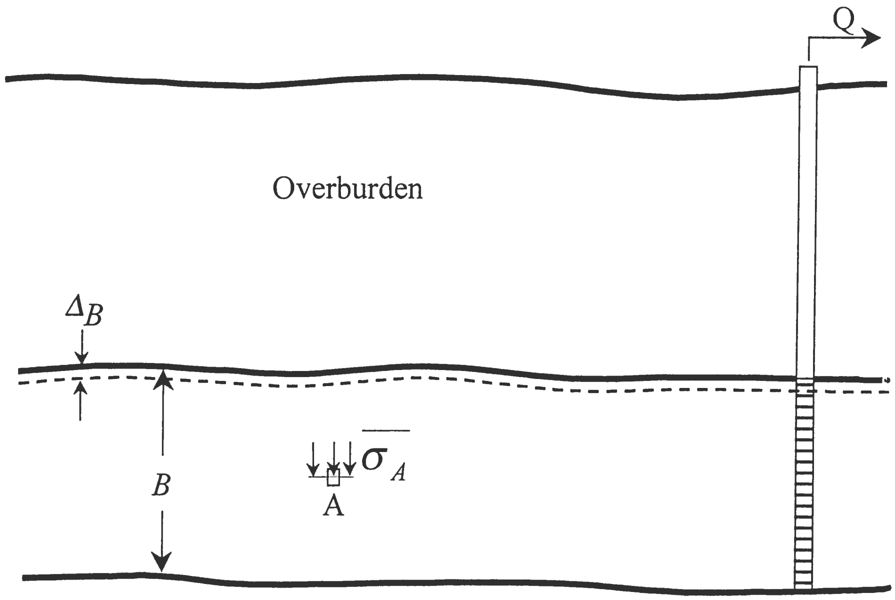

Figure 2 - Height, Settlement and State of Stress in an Aquifer

Figure 2 shows a soil profile consisting of overburden above an aquifer of thickness B. The settlement ΔB of the aquifer is determined by the strain of the layer,

$$\varepsilon = \frac{\Delta B}{B} = \frac{\Delta V}{V} = \frac{\Delta V_v}{V_s + V_v} = \frac{\Delta V_v / V_s}{V_s / V_s + V_v / V_s} = \frac{\Delta e}{1+e}. \qquad \text{Eq. 7}$$

Using this definition of the strain, the settlement equals

$$\Delta B = \frac{B \Delta e}{1+e} \qquad \text{Eq. 8}$$

This is the standard form of the one-dimensional consolidation equation given in all soil mechanics textbooks.

The layers deform because a change of effective stress occurs over time. Terzaghi (1925) defined the effective stress $\bar{\sigma}$ as the difference between the total stress that is developed at a point σ, and in this case is the sum of all the weights of the overburden soil, rock and water above the unit square of interest, and the water pressure, u, at that point,

$$\bar{\sigma} = \sigma - u. \qquad \text{Eq. 9}$$

The effective stress is significant because the behavior of the soil and rock responds to changes in effective stress, not total stress. Stated another way, the material "feels" the effective stress, not the total stress. Changes in total stress are only significant in that they may change the effective stress if the water pressure otherwise remains constant.

The change in void ratio is related to the applied effective stress by the coefficient of compressibility, a_v, which is the ratio of change in void ratio divided by the corresponding change in effective stress as is shown in Fig. 3.

$$a_v = -\frac{\Delta e}{\Delta \overline{\sigma}} \quad \text{Eq. 10}$$

or, solving for Δe,

$$\Delta e = -a_v \Delta \overline{\sigma} \quad \text{Eq. 11}$$

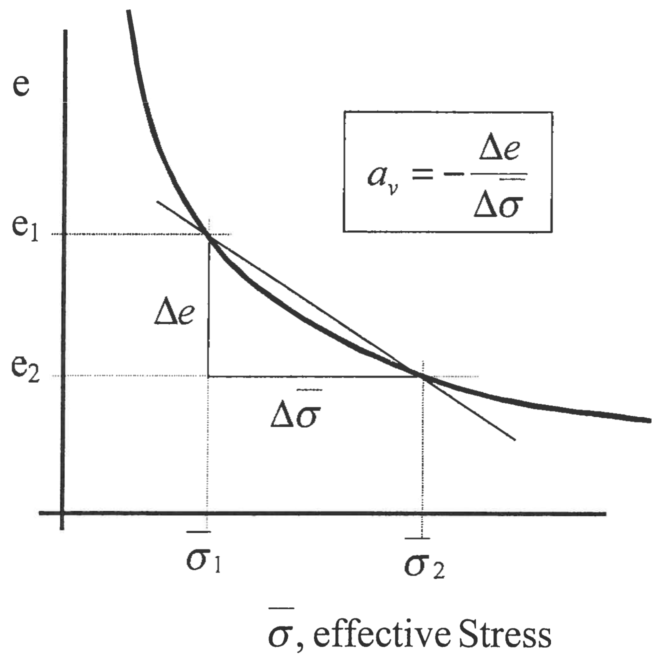

Figure 3 - Coefficient of Compressibility is

Slope of Void Ratio vs. Effective Stress Curve

This coefficient is typically determined in the laboratory, but may be determined in the field by indirect methods.

The change in void ratio at a point "A" in Fig. 2 can be determined between two points in time when the piezometeric pressures are known or can be estimated. In most cases, the most

significant times would be before and after the pumping has occurred. Using two arbitrary times t_1 and t_2, there would be corresponding effective stresses $\bar{\sigma}_1$ and $\bar{\sigma}_2$.
The change in effective stress is given by

$$\Delta\bar{\sigma} = \bar{\sigma}_2 - \bar{\sigma}_1 = (\sigma_2 - u_2) - (\sigma_1 - u_1) = (\sigma_2 - \sigma_1) - (u_2 - u_1). \qquad \text{Eq. 12}$$

Because the total stress is the sum of the weights above the point of interest, it is determined by adding the products of the unit weights and the layer thicknesses for all the layers. While there have been large drawdowns in the coal seams, data has been presented which indicates that there has not been much if any water pressure changes in the overlying layers. This is because the coals are generally over and underlain by shale beds which can effectively retard the flow of water from other layers. Therefore, the water levels have not dropped in the upper layers with the consequence that the total stress at point "A" has not changed. Equation 12 is then written

$$\Delta\bar{\sigma} = -(u_2 - u_1) \qquad \text{Eq. 13}$$

indicating that the change in effective is given by the changes in water pressure at the two times.

This can be related to the piezometric heads since

$$\phi = Z + u/\gamma_w \qquad \text{Eq. 14}$$

which states that the total piezometric head is equal to the elevation head Z and the pressure head, u/γ_w, where u is the water pressure and γ_w is the unit weight of water. Solving for u gives

$$u = (\phi - Z)\gamma_w \qquad \text{Eq. 15}$$

Substituting this back into Equation 13 yields

$$\Delta\bar{\sigma} = -(u_2 - u_1) = -[(\phi_2 - Z_2) - (\phi_1 - Z_1)]\gamma_w \qquad \text{Eq. 16}$$

At any given location, the elevation is constant, so Eq. 16 can be reduced to

$$\Delta\bar{\sigma} = -(\phi_2 - \phi_1)\gamma_w = -\gamma_w \Delta\phi \qquad \text{Eq.17}$$

which states that the change in effective stress is equal to the product of the unit weight of water and the difference in piezometric heads at two different times. Now the change in void ratio given in Eq. 11 can be related to the change in head in Eq. 17,

$$\Delta e = -a_v \Delta \overline{\sigma} = a_v \gamma_w \Delta \phi \qquad \text{Eq. 18}$$

This can be related back to the change in porosity in Eq. 5 to give

$$\Delta n = \frac{\Delta e}{1+e} = \frac{a_v \gamma_w}{1+e} \Delta \phi \qquad \text{Eq. 19}$$

This equation is also the equation for one-dimensional strain, defined in Eq. 7.

The Degree of Saturation used in Eq. 3 can be related back to the operation of the wells. If a portion of the aquifer around the well becomes unsaturated, the flow into the well would be restricted and the pumping rate would decrease. Therefore, operationally, the aquifer will always be kept saturated and $S_a = 1$. The second term on the right hand side of Eq. 3 goes to zero because S_a is a constant. Finally, Eq. 3 can be written

$$\frac{\partial S_a n}{\partial t} \cong S_a \frac{\Delta n}{\Delta t} + n \frac{\Delta S_a}{\Delta t} = (1)\frac{\Delta n}{\Delta t} + n \frac{\Delta(1)}{\Delta t} = \frac{a_v \gamma_w}{1+e} \frac{\Delta \phi}{\Delta t} \qquad \text{Eq. 20}$$

Finally, in the limit as time approaches zero, Eq. 20 becomes

$$\frac{\partial S_a n}{\partial t} = \frac{a_v \gamma_w}{1+e} \frac{\partial \phi}{\partial t} = S_s \frac{\partial \phi}{\partial t} \qquad \text{Eq. 21}$$

In this form, the coefficient is defined as the Specific Storage of the aquifer, and is a measure of the volume of water released from an aquifer due to a unit decrease in head. With this observation, the aquifer deformation ΔB described in Eq. 8 may be combined with Eqs. 19 and 21 to give

$$\Delta B = \frac{B \Delta e}{1+e} = \frac{B a_v \gamma_w}{1+e} \Delta \phi = B S_s \Delta \phi = S \Delta \phi \qquad \text{Eq. 22}$$

which leads to the simple statement that the settlement of the aquifer is equal to the Storage Coefficient times the change in head in the aquifer over time. The Storage Coefficient, equal to the product of the Specific Storage and the aquifer thickness, is a commonly determined value in aquifer testing. It can be determined using the standard groundwater hydrologic techniques used for any aquifer.

Another formulation for Specific Storage gives additional insight into its evaluation. The Specific Storage may be written as

$$S_s = \gamma_w n \beta + \frac{\gamma_w}{E_{aquifer}} \qquad \text{Eq. 23}$$

The reciprocal Bulk Modulus of water is a tabulated value and equals approximately

$$\beta = 2.2x10^{-8} \ \frac{1}{psf}$$

Using values that are typical for a shallow aquifer with an average depth of 50 feet, the porosity $n = 0.4$ and $E_{soil} = 1{,}000{,}000$ psf, then

$$S_s = \gamma_w n \beta + \frac{\gamma_w}{E_{aquifer}} = (62.4pcf)(0.4)\left(2.2x10^{-8} \ \frac{1}{psf}\right) + \frac{62.4pcf}{1{,}000{,}000psf}$$

$$S_s = 5.4x10^{-7} \ \frac{1}{ft} + 6.24x10^{-5} \ \frac{1}{ft}$$

$$S_s = 6.29x10^{-5} \ \frac{1}{ft}$$

This result shows that the compressibility of water is not a significant factor in aquifer deformation or supply. As the depth of the aquifer increases, the modulus of elasticity increases while the reciprocal bulk modulus decreases slightly. These both indicate that the compressibility of the aquifer decreases with increasing depth. This is why most regional settlement problems occur in situations with shallow aquifers or sedimentary deposits close to the ground surface.

List of Symbols

a_v	Coefficient of Compressibility, ratio of the change in void ratio to the change in effective stress.
B	Thickness of a geologic layer, especially the thickness of an aquifer.
e	Void ratio, ratio of volume of voids to volume of solid particles
E_{soil}	Modulus of Elasticity of the soil.
K_x, K_y, K_z	Hydraulic Conductivity of the aquifer in the x, y, and z directions. Also referred to as the Coefficient of Permeability.
n	Porosity, ratio of void volume to total volume in an aquifer.
S	Storage Coefficient, the product of the Specific Storage, S_s and the thickness of the aquifer, B
S_a	Degree of Saturation, ratio of the volume of water to the volume of void.
S_s	Specific Storage, amount of water released or gained from a unit volume due to a unit change in head.
S''	Source or sink of water inside the element which is not associated with flow
t	Time
u	Pressure of water.
x, y, z	Cartesian coordinates
β	Reciprocal Bulk Modulus of Water, equal to $2.2\text{x}10^{-8}\ \frac{1}{psi}$
ϕ	Total head or Piezometric head of water, equal to the elevation and pressure heads of the water. A measure of the energy of the water.

σ	Total stress acting at a point due to the weight of the overburden soil, rock and water.
$\overline{\sigma}$	Effective Stress, the difference between the total stress and the water pressure. The soil deforms due to changes in the effective stress.